

**On the “circular vacuum noise” in electron storage rings**Haret C. Rosu<sup>1</sup>Instituto de Física de la Universidad de Guanajuato, Apartado Postal E-143, León,  
Guanajuato, México**Abstract**

We discuss the proposal of Bell and Leinaas (BL) to measure the circular Unruh effect in storage rings. The ideal concept ‘circular Unruh effect’ has a more realistic correspondent such as ‘circular vacuum noise’ used by Shin Takagi [Prog. Theor. Phys. Suppl. 88, 1 (1986)]. The BL resonance behavior does not fit to the SPEAR first order betatron resonance at 3.605 GeV, but of course, the real experimental situation is much more complicated, corresponding, as a matter of fact, to an even more general term that one may call ‘synchrotron noise’. In the final section we focus on radiometric aspects of storage ring physics, such as the problem of establishing better quantum field radiometric standards at high energies. The Unruh-like effect could be a useful guide for that purpose.

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**1 Introduction**

About thirty years ago Sokolov and Ternov showed that electrons circulating in a vertical and uniform magnetic field get polarized [1]. This occurs because in the quantum synchrotron emissions there is a very small spin-flip power( only  $10^{-11}$  of the classical continuous power), which is accumulating over a timescale of tens of minutes to a few hours to give a final asymptotic polarization  $P_{lim} = 8/(5\sqrt{3}) = 0.924$ . This number found three decades ago as the result of a simple theoretical QED exercise is today a famous figure of accelerator physics. In the seventies, Derbenev and Kondratenko [2] obtained a more realistic formula for the limiting beam polarization containing the spin-orbit coupling function. This vector parameter expresses

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the correlations among the positions of the synchrotron emission events on the orbit and the spin motion (fluctuations of the precession axes). It is completely determined by the magnetic lattice of the accelerator. In a certain sense we could say that the synchrotron vacuum fluctuations are angularly constrained by the magnetic boundaries of the guiding structure via the spin axis of the electrons (a kind of magnetic orientational Casimir effect).

The Sokolov-Ternov effect has been confirmed at the major synchrotrons and storage rings of the world. Even at LEP, the largest storage ring at the moment, a Compton polarimeter detected some polarization [3]. Also transverse beam polarizations have been measured at HERA by means of a laser polarimeter [4]. However, the whole spin game becomes technically interesting because of the depolarizing resonances (DRs) that one encounters easily at any high energy circular accelerator. In a way, accelerator DRs are similar to the classical resonances of the Solar System, asteroid belt and planetary rings, of course at much different scales of time and space. The accelerator DR's are a consequence of the anomalous (presumably irrational) magnetic moment of the electron and the proton ( $a = \frac{g-2}{2}$ ;  $a_e = 0.00116$ ,  $a_p = 1.793$ ). DRs have been classified according to their cause and their parameters have been determined with great accuracy. The point is that at high energies the density of DRs becomes embarassingly large, a fact of great concern for the accelerator physicists.

## 2 Can one see the BL effect ?

The DK polarization formula of 1973 is overwhelmingly used by accelerator people. It includes in a well-established way the effects of DRs on the equilibrium polarization. This formula reads

$$P_{eq}^{DK} = \frac{8}{5\sqrt{3}} \frac{\langle |\rho|^{-3} \hat{b}(\hat{n} - F_{DK}) \rangle}{\langle |\rho|^{-3} (1 - 2/9(\hat{n} \cdot \hat{v})^2 + 11/18 |F_{DK}|^2) \rangle}, \quad (1)$$

where  $F_{DK} = \gamma \frac{\partial \hat{n}}{\partial \gamma}$  is the spin-orbit coupling function, which takes into account the depolarizing effects of jumps between various trajectories differing from the reference closed orbit,  $\rho$  is the bending radius,  $\hat{b}$  a unit vector along the transverse magnetic field component. The brackets indicate an average over the ring circumference and over the ensemble of particles in the beam. The unit vector  $\hat{n}$  is the time-independent spin solution of the BMT equation, attached to each particle trajectory.

On the other hand, motivated by Unruh thermal-like effect, which can show up in circular motion too (see section 3), Bell and Leinaas [5] developed a formalism which is closer to quantum field theory and therefore more acceptable by theoretical physicists. Aiming at very peculiar effects, first of all, the correct way of taking into account vertical electron recoils, Bell and Leinaas have been forced to consider the case when the spin-orbit function is zero. This is valid only for perfectly aligned weak focusing storage rings for which the accelerating fields are independent of arc length and the magnetic field is vertical on the perfectly planar closed orbit with a small magnetic gradient  $n = -(B/R)^{-1}(\partial B/\partial r)$ . The meaning of  $r$  is the radial displacement from the closed orbit  $R$ . Usually the vertical betatron fluctuations as determined by the horizontal synchrotron emissions are a negligible effect as compared to the more common accelerator stochastic excitations. However, for weak focusing machines such effects expressed by Bell and Leinaas in terms of a parameter  $f$  have also a linear contribution to the limiting polarization and become predominant over the main part of the stochastic excitations included in the spin-orbit function of DK. Moreover, in an ideal perfectly aligned machine the DK spin-orbit function is actually zero. The corresponding equilibrium betatron emittance of the vertical oscillations is calculated by BL explicitly by means of the standard Lorentz-Dirac equation in order to take into account the radiation damping (on the Langevin character of the Lorentz-Dirac equation we shall comment in a future work, [6]). The final BL polarization formula is

$$P_{BL} = (8/5\sqrt{3}) \times \frac{1 - f/6}{1 - f/18 + 13f^2/360} , \quad (2)$$

where the BL parameter  $f$  is given by

$$f = \frac{2}{\gamma} \times \frac{\nu_s Q_\beta^2}{Q_\beta^2 - \nu_s^2} . \quad (3)$$

In Eq.(3)  $\gamma$  is the relativistic kinematical factor,  $\nu_s$  is the spin tune, ( $\nu_s = a_e \gamma = E/0.441(GeV)$ ), and  $Q_\beta^2$  is the betatron tune.

The BL formula has the usual intrinsic betatron resonance condition  $\nu_s = Q_\beta = \sqrt{n}$ . However the  $f$  parameter implies a more intricate behavior of the polarization in passing through the resonance, which should be isolated and of first order, namely  $P_{eq}$  drops from 0.924 to -0.169,

then increases to 0.992 to fall eventually to 0.924. A thorough comparison of the DK and BL formalisms have been provided by Barber and Mane [7]. At the same time Mane has generalized the BL result to the case of strong-focusing storage rings (see formula (41) in ref. [7]). The  $f$  parameter turns now to be a vector quantity defined as  $\mathbf{f} = -(2/\gamma)\partial\mathbf{n}/\partial\beta_b$  where again the subscript  $b$  denotes the direction of the magnetic field, not necessarily vertical. This allows a coupling of the horizontal betatron and synchrotron oscillations to the vertical fluctuations;  $\mathbf{n}$  is the chosen spin quantization axis. The only first order resonance at which one might think of a BL effect on the SPEAR polarization data [8], is the resonance  $\nu_s = 3 + \nu_y$  at  $E = 3.605$  GeV, but we have to emphasize that the experimental conditions are very different from those required by the ideal BL case. The SPEAR  $E = 3.605$  GeV resonance would correspond to  $n=67$ . This is already a rather high  $n$  to make the BL effect unobservable. At SPEAR energies we encounter a well developed forest of resonances not the isolated BL situation. Also SPEAR has a superperiodicity of two for which all the odd resonances are forbidden. Their presence is due only to higher order effects, making them very narrow, (for an interpretation in terms of nonlinear tunespreads see [9], [10]).

### 3 More about circular vacuum noise

We now come to the problem of interpretation. Bell and Leinaas were motivated in their particular treatment of fluctuations in electron storage rings by the chance of revealing Unruh effect of vacuum fluctuations. As it is well known there is a close parallel between Hawking effect and Unruh effect [11]. Their proposal was chronologically the second one to detect such fundamental effects within terrestrial laboratories after that of Unruh [12], who developed a hydrodynamical analogy for Hawking effect. It has been considered as the most feasible one for a long time. However, in their 1987 paper BL are aware of the difficulty of introducing a temperature parameter for synchrotron fluctuations. We recall that for one parameter problems, like Schwarzschild black holes and Rindler linear accelerated motion, the quantum field vacuum turns formally into an equilibrium thermal state. One may introduce a thermodynamic temperature directly related to that one parameter of the problem, the Schwarzschild mass in the first case, the constant

proper acceleration in the latter. There are papers connecting the usual QED bremsstrahlung and zero-energy Rindler photons [13].

However, the situation is much different in the uniform circular motion, and not only because there is no horizon, but more important because the acceleration, despite being a constant, has the rather peculiar form given by

$$\alpha_c = \frac{\rho\omega_0^2}{1 - \rho^2\omega_0^2} . \quad (4)$$

Here  $\rho$  is the radius of the orbit and  $\omega_0$  is the cyclotron frequency. Units such that  $\hbar = c = 1$  are used.

The point is that in the circular motion the vacuum energy density cannot be written in the canonical Planck spectrum form [14]. Since the Fourier transforms of the Wightman functions have singularities of the branch cut type (implying  $\Theta$  functions) one will find out an energy density of the vacuum fluctuations as a sum over cyclotron harmonics. The quantum zero-point noise is in this case of an intermediate nature between an additive and a multiplicative noise. We shall address this very interesting problem in another work [6].

The vacuum energy density of a massless scalar field in the circular case can be written as follows [14]

$$\frac{de}{d\omega} = \frac{\omega^3}{\pi^2} \left\{ 1/2 + \frac{\omega_0}{\gamma\omega} \sum_{n=0}^{n=\infty} \frac{\beta^{2n}}{2n+1} \sum_{k=0}^n (-1)^k \frac{n-k-\frac{\omega}{\gamma\omega_0}}{k!(2n-k)!} \Theta(n-k-\frac{\omega}{\gamma\omega_0}) \right\} . \quad (5)$$

From Eq.(5) one can see that to a certain power of the velocity many vacuum cyclotron harmonics could contribute making the energy density spectrum a quasi-continuous one but with different shape and scale as compared with a pure Planckian spectrum.

The naïve introduction of the circular-motion acceleration  $\alpha_c = \gamma^2\omega_0 v$  into the Planckian function  $\{\exp[2\pi\frac{\omega v}{\gamma^2\omega_0}] - 1\}^{-1}$  shows an essential singularity at  $v=0$  which does not allow an expansion in velocity powers.

Hacyan and Sarmiento [15] developed a formalism, very close to the scalar case, for calculating the vacuum stress-energy tensor of the electromagnetic field in an arbitrarily moving frame and applied it to a system in uniform rotation. They provided formulas for energy density, Poynting flux, and stress of the zero-point field in such a frame. Moreover, Mane [16] has suggested the

Poynting flux of Hacyan and Sarmiento to be in fact synchrotron radiation when coupled to an electron.

The relationship of circular noise and Rindler noise has been under focus in the works of Letaw [29], Gerlach [18], and Takagi [19]. These authors managed to determine an exact connection for the massless scalar field between the circular noise and so-called “drifted Rindler noise” seen by a quantum detector which is uniformly accelerated but has also a constant speed in the direction perpendicular to the acceleration. It is just the limiting case of infinite radius for circular uniform motion. In the three- dimensional Minkowski space the circular motion is a helix winding around the time axis. The Rindler trajectory is also a helix this time winding with an imaginary pitch around a space axis. They are examples of the six classes of stationary motions obtained by Letaw [29] and possessing stationary noises. Nevertheless these stationary noises do not satisfy the KMS condition, i.e., the principle of detailed balance in quantum field theory [6]. To quote Takagi [19], “the effective temperature depends on the energy and does not provide a very useful concept.”

## 4 On radiometry at storage rings

The considerations in the previous section make us expound a little on the (primary) radiometric standards at high energies. As it is well known the common blackbody radiators are limited to  $3 \times 10^3$  K for technical reasons. The idea to use electron storage rings with their magnetobrehmsstrahlung/synchrotron spectrum as primary radiometric standards at high energies is not new, and important contributions have been made in the past [20]. Already 4 electron storage rings working at 800 MeV, approximately, are used as radiometric standard sources: SURF II, VEPP 2M, TERAS, and BESSY. The projected BESSY II will also be designed with regard to radiometric applications [21]. The primary radiometric character of the synchrotron spectrum is based on the famous spectral photon flux formula of Schwinger [22].

If the comparison between the (non-thermal) magnetobrehmsstrahlung spectral distribution and the (thermal) black body one is made by identifying their two maxima [23], one concludes that for a beam of 1 GeV the “temperature” of the maximum of the brehmsstrahlung (syn-

chrotron) radiation is about  $10^7$  K. It is a gain of 4 orders of magnitude for the important field of radiometry. Nevertheless, as has been argued in section 3 above, we are not in completely equivalent situations. The shapes of the two spectral densities are different, i.e.,

$$f(y) = (9\sqrt{3}/8\pi)y \int_y^\infty K_{5/3}(x)dx \quad (6)$$

for the synchrotron radiation, as compared to

$$\phi(y) = (15/\pi^4) \frac{y^3}{e^{-y} - 1} \quad (7)$$

for the blackbody radiation. In Eqs.(6) and (7) the  $y = \frac{\omega}{\omega_{max}}$  variable is a scaled frequency, where  $\omega_{max}$  is corresponding to the maximum of the synchrotron radiation  $\omega_{max} = \omega_c \gamma^3$ ,  $\omega_c$  being the cyclotron angular frequency.

One may hope to obtain non-thermal distributions from a thermal one by means of q-deformations [24]. Indeed, the features of the emitted spectrum are strongly dependent on the electron-photon interactions along the beam trajectory and a q-boson interpretation seems natural.

In the search for a better primary radiometric standard we recall that a formal truly thermal ambience such as the Unruh one is not a local property of the trajectory, but a global one [25]. We need a constant power spectrum uniformly distributed over the radiation spectrum of the electron. To get this an undulator of a special type made of a collection of so-called “short magnets” [26] is needed. For a single “short magnet”, it has been shown [26] that the electron will radiate a white noise with the angular frequencies distributed from zero up to the frequency  $\omega_{short} = (c/l)\gamma^2$ . Here  $l = 2\alpha R$  is the arclength of the electron trajectory in the “short magnet”. The criterion for a magnet to be a “short magnet” is  $\alpha \ll \frac{mc^2}{E}$ . They shift the maximum of the synchrotron radiation to longer wavelengths, and depending on their number we could move the maximum of the synchrotron spectrum to whatever wavelength we like in the spectral width of the quasi-white noise.

It seems therefore that various types of insertion devices will be of great help in the field of radiometry [27]. Especially for storage rings working at higher energies (ESRF, Spring 8, APS) detailed models for the radiation at the insertion devices are required for its use in radiometry.

A highly interesting field regarding the connections between the coherence and polarization features of the insertion devices and radiometry is foreseeable (see [27] for a first step in this direction). It is known that even such a good simulator of blackbody radiation as cavity radiation has a spatial frequency spectrum, which is not white over the real plane waves [28].

It is worth noting also that high-energy radiometry could be done in the future for other types of brehmsstrahlungs, e.g., the beamstrahlung at linear colliders.

Radiation noises and radiometry at colliders are obviously connected with chaos problems. Because in the phase space of a system possessing  $N$  dof's the uncertainty principle replaces every portion of the continuum of classical trajectories of states in each  $\hbar^N$  volume cell by only one, so-called quantum state, one immediately concludes that the physical quantum chaos is more ordered than the classical (geometric) one. The latter is only a limiting pure mathematical chaos from the point of view of real, physical measurement. But if one will insist to go beyond quantum theory of physical measurements a knowledge of the topology and geometry of the proximity of points is unavoidable [29].

## 5 Concluding Remarks

We presented a discussion of the ‘circular Unruh’ effect put forth by Bell and Leinass [5]. Recently, Cai, Lloyd, and Papini [30] claimed that the spin-rotation-gravity coupling, also known as Mashhoon effect in gravitation theory [31], is practically stronger than the acceleration effect at all available energies, but the comparison is rather difficult and not so direct. Another interesting effect is mentioned by Fröhlich and Studer [32] for the case of non-electronic storage rings. A beam of non-relativistic particles with spin display a variant of the Einstein-de Haas effect which could show up as a tidal Zeeman energy affecting, after relaxing to a steady state, the ratio of spin-up to spin-down ions in the beam.

Finally, we sketched above some radiometric aspects of storage ring physics that may be useful for future detailed analyses.



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